DIRECTION OF ARRIVAL ESTIMATION IN WIRELESS MOBILE COMMUNICATIONS USING MINIMUM VERIANCE DISTORSIONLESS RESPONSE

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ABSTRACT

This paper presents a tool for the modelling, analysis, and simulation of direction-of-arrival (DOA) estimation in wireless mobile communication systems utilizing adaptive antenna arrays. The developed tool implements the Minimum Variance Distortionless Response (MVDR) algorithm. To demonstrate the versatility and accuracy of the developed tool, it is used to carry out a performance study of the MVDR algorithm by investigating the effect of various parameters related to the signal environment and sensor array.

1. INTRODUCTION

Adaptive signal processing sensor arrays, known also as smart antennas, have been widely adopted in third-generation (3G) mobile systems because of their ability to locate mobile users with the use of DOA estimation techniques. Adaptive antenna arrays also improve the performance of cellular systems by providing robustness against fading channels and reduced collateral interference [1].

The goal of direction-of-arrival (DOA) estimation is to use the data received on the downlink at the base-station sensor array to estimate the directions of the signals from the desired mobile users as well as the directions of interference signals. The results of DOA estimation are then used by to adjust the weights of the adaptive beamformer so that the radiated power is maximized towards the desired users, and radiation nulls are placed in the directions of interference signals. Hence, a successful design of an adaptive array depends highly on the choice of the DOA estimation algorithm which should be highly accurate and robust.

This paper investigates the Minimum Variance Distortionless Response (MVDR) algorithm for DOA estimation. It starts by carrying out an analysis of the algorithm starting from a realistic signal model and then develops a simulation tool which is used to carry out a performance study of the algorithm. This includes investigating the effect of parameters related to the signal environment such as the number of incident signals and their angular separation. It also investigates effect of parameters related to the design of the sensor array itself including number of array elements and their spacing.

2. DOA ESTIMATION USING MVDR ALGORITHM

The MVDR algorithm involves estimating the noise subspace from the correlation matrix on which the M array steering vectors are projected. These steering vectors are also known as direction vectors and they represent the response of an ideal array to the signal sources. The signal sources can be derived from the direction vectors which are as orthogonal to the noise subspace.

The algorithm starts by constructing a real-life signal model. Consider a number of plane waves from M narrow-band sources impinging from different angles θ_i, $i = 1, 2, \ldots, M$, impinging into a uniform linear array (ULA) of N equi-spaced sensors, as shown in Figure 1.

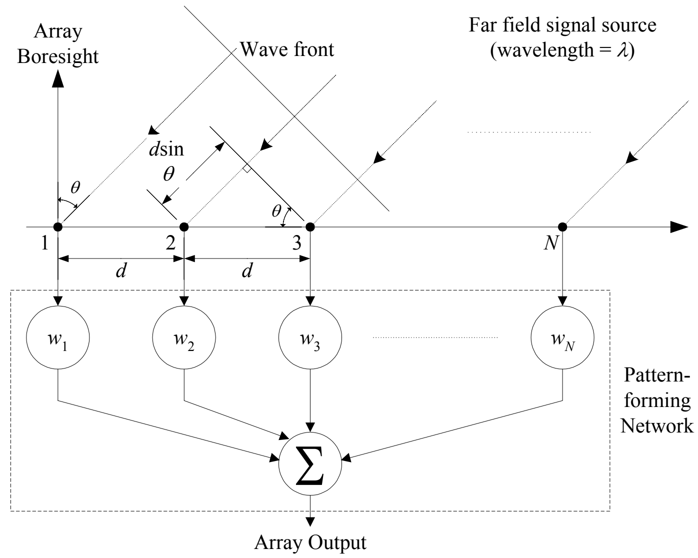


Figure 1: A plane wave incident on a uniform linear array of *N* equi-spaced sensors [2].

At a particular instant of time *t*, *t*=1,2, …, *K*, where *K* is the total number of snapshots taken, the array output will consist of the signal plus noise components. The signal vector $\mathbf{x}(t)$ can be defined as [3]:

$$\mathbf{x}(t) = \sum_{m=1}^{M} \mathbf{a}(\theta_m) \cdot \mathbf{s}_m(t) \tag{1}$$

where $\mathbf{s}(t)$ is an $M\times 1$ vector of source waveforms, and for a particular source at direction θ from the array boresight; $\mathbf{a}(\theta)$ is an $N\times 1$ vector referred to as the array response to that source or array steering vector for that direction. It is given by:

$$\mathbf{a}(\theta) = [1 \quad e^{-j\phi} \quad \ldots\ldots \quad e^{-j(N-1)\phi}]^T \tag{2}$$

where T is the transposition operator, and ϕ represents the electrical phase shift from element to element along the array. This can be defined by:

$$\phi = (2\pi / \lambda)\, d \cos\theta \tag{3}$$

where d is the element spacing and λ is the wavelength of the received signal.
The signal vector $\mathbf{x}(t)$ of size $N\times 1$ can be written as:

$$\mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t) \tag{4}$$

where $\mathbf{A}=[\mathbf{a}(\theta_1) \ldots \mathbf{a}(\theta_M)]$ is an $N\times M$ matrix of steering vectors.
The array output consists of the signal plus noise components, and it can be defined as:

$$\mathbf{u}(t) = \mathbf{x}(t) + \mathbf{w}(t) \tag{5}$$

where $\mathbf{x}(t)$ and $\mathbf{w}(t)$ are assumed to be uncorrelated and $\mathbf{w}(t)$ is modelled as temporally white and zero-mean complex Gaussian process. Equation 5 can be written in matrix form of size $N\times K$ as:

$$\mathbf{U} = \mathbf{A} \cdot \mathbf{S} + \mathbf{W} \tag{6}$$

where $\mathbf{S}=[\mathbf{s}(1) \ldots \mathbf{s}(K)]$ is an $M\times K$ matrix of source waveforms and $\mathbf{W}=[\mathbf{w}(1) \ldots \mathbf{w}(K)]$ is an $N\times K$ matrix of sensor noise. The spatial correlation matrix $\mathbf{R}$ of the observed signal vector $\mathbf{u}(t)$ can be defined as:

$$\mathbf{R} = E[\mathbf{u}(t) \cdot \mathbf{u}(t)^H] \tag{7}$$

where $E[\ ]$ and H are the expectation and conjugate transpose operators, respectively. Substituting (5) into (7), the spatial correlation matrix $\mathbf{R}$ can now be expressed as:

$$\mathbf{R} = E[\mathbf{A} \cdot \mathbf{s}(t) \cdot \mathbf{s}(t)^H \cdot \mathbf{A}^H] + E[\mathbf{w}(t) \cdot \mathbf{w}(t)^H] \tag{8}$$

For this signal model, the correlation matrix $\mathbf{R}$ will have M signal eigenvalues, and N-M noise eigenvalues. Let $\mathbf{E}_s$ be the matrix constructed of the corresponding M signal eigenvectors $\mathbf{E}_s=[e_1\ e_2 \ldots e_M]$, and $\mathbf{E}_n$ be the matrix containing the remaining N-M noise eigenvectors $\mathbf{E}_n=[e_{M+1}\ e_{M+2} \ldots e_N]$. The peaks in the MVDR angular spectrum occur whenever the steering vector $E(\phi)$ is orthogonal to the noise subspace.This technique minimizes the contribution of the undesired interferences by minimizing the output power while maintaining the gain along the look direction to be constant, usually unity. That is,

$$\min E[|\, y(\theta)\,|^2] = \min \mathbf{w}^H \mathbf{R}_{uu} \mathbf{w} \quad , \quad \mathbf{w}^H A(\theta_0) = 1 \tag{9}$$

Using Lagrange multiplier, the weight vector that solves equation (1) can be shown to be:

$$w = \frac{\mathbf{R}_{uu}{}^{-1} A(\theta)}{A^H(\theta) \mathbf{R}_{uu}{}^{-1} A(\theta)} \tag{10}$$

Now the output power of the array as a function of the DOA estimation, using MVDR beamforming method [4], is given by MVDR spatial spectrum as,

$$P_{MVDR}(\theta) = \frac{1}{A^H(\theta) \mathbf{R}_{uu}{}^{-1} A(\theta)} \tag{11}$$

The angles of arrival are estimated by detecting the peaks in this angular spectrum.

3. GRAPHICAL USER INTERFACE

The MVDR algorithm has been implemented using MATLAB version 6.5. A GUI has also been built to ease the simulation. A layout of the GUI is depicted in Figure 2. The user can input the signal parameters including the number of snapshots *K*, the number of mobile users *M*, and their angle(s) of arrival θ_i. This information is used to generate a realistic signal model. As for the sensor array, the user may input the number of array elements *N* and their spacing *d*. Default values for these parameters can be retrieved at any time by pressing the "Default" button. The simulation can be started by clicking on the "Run" button. This will produce a plot of the MVDR angular spectrum. This plot can be saved to a file by pressing the "Save" button. Alternatively, the plot may be sent to a printer by pressing the "Print" button.

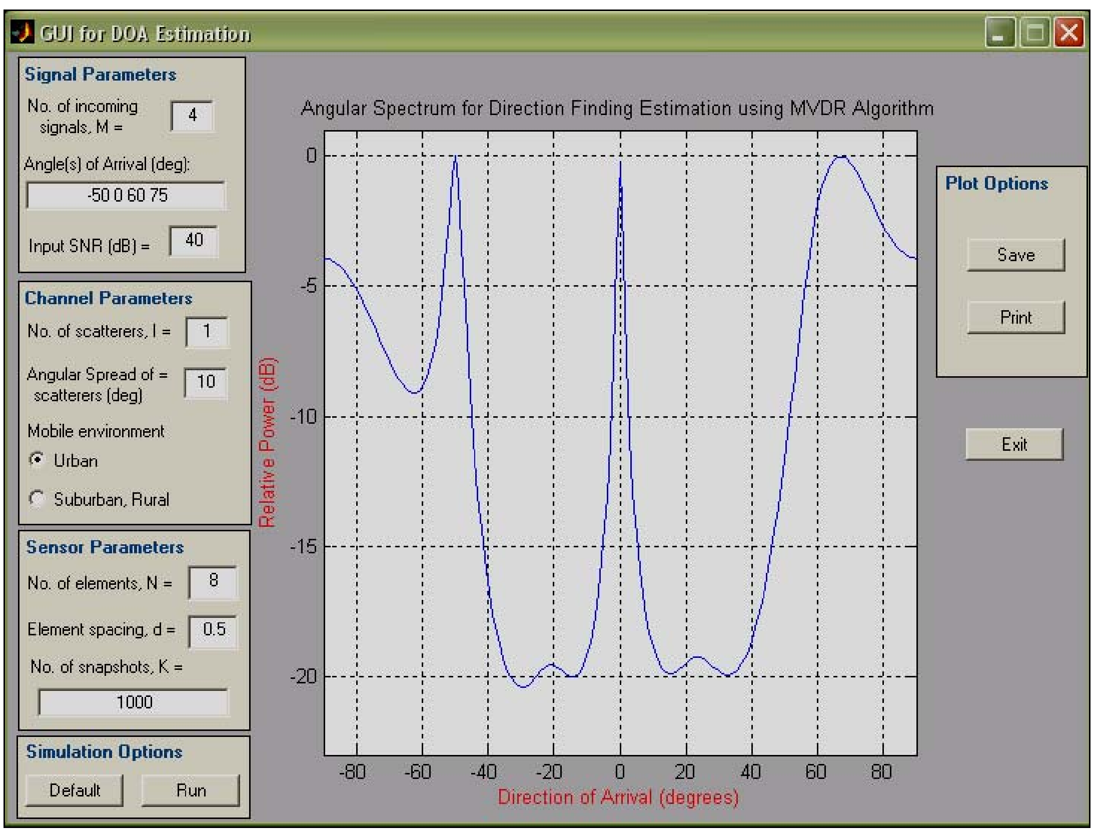


Figure 2: Layout of graphical user interface.

4. PERFORMANCE STUDY

To demonstrate the versatility and accuracy of the developed tool, it is used to study the effect of changing a number of parameters related to the signal environment as well as the sensor array.

4.1 Number of Incident Signals from Mobile Users

Figure 3 shows the MVDR angular spectrum generated due to signals arriving from *M*=1 user and *M*=7 users. When there is only one mobile user (*M*=1) is present in the vicinity of the base station, the MVDR algorithm performs better since it produces an angular spectrum with a sharp peak and a lower noise floor. The performance of the algorithm degrades when there are many mobile users because the spatial correlation between the incoming signals makes it difficult for the algorithm to resolve them successfully.

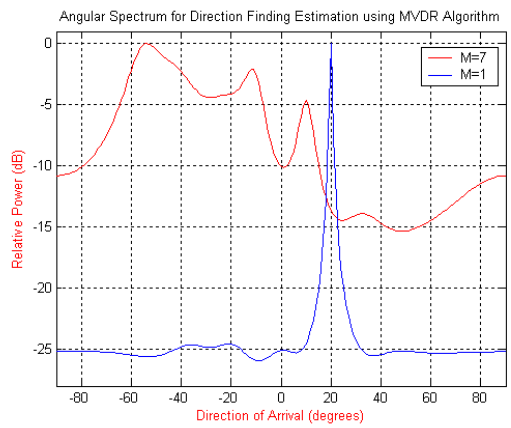
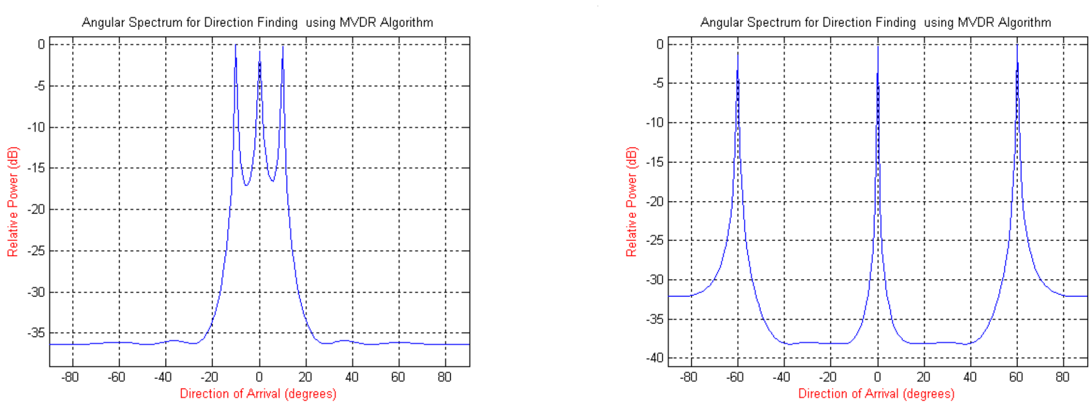


Figure 3: Effect of number of mobile users *M* on MVDR angular spectrum.

4.2 Angular Separation of Incident Signals from Mobile Users

When the mobile users are close to each other, there will be a strong correlation between the signals. In this case the MVDR algorithm finds it difficult to resolve the users. This is illustrated in Figure 4(a) for 3 adjacent users. However, the performance improves significantly as the users move away from each other, as shown in Figure 4(b) for which the MVDR angular spectrum has sharper peaks and lower noise floor.

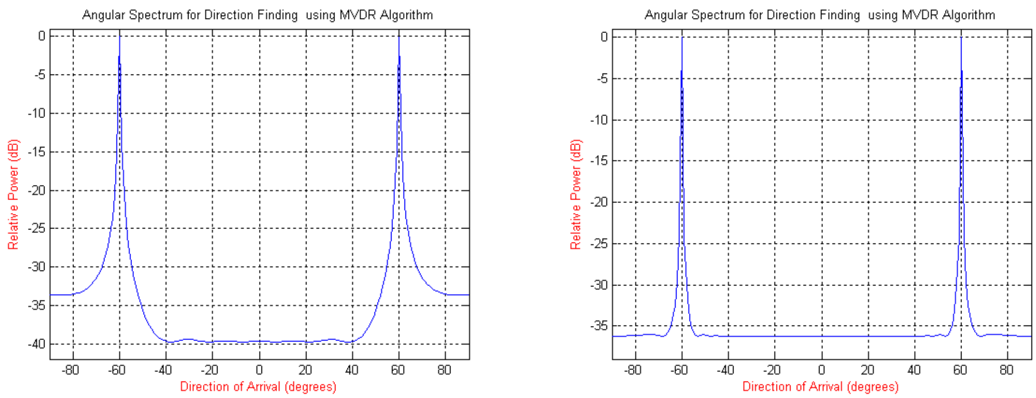


(a) **(b)**

Figure 4: Effect of angular separation of mobile users on MVDR spectrum: (a) adjacent users, and (b) separated users.

4.3 Number of Elements in the Sensor Array

Figures 5(a) and 5(b) show the MVDR angular spectrum using an eight-element array (*N*=8) and three-element array (*N*=3), respectively. It is evident that using more elements improves the resolution of the algorithm in detecting the incoming signals. This is achieved, however, at the expense of computational efficiency and hardware complexity of the sensor array.



(a) **(b)**

Figure 5: Effect of number of array elements *N* on MVDR spectrum:
(a) *N*=8elements, and (b) *N*=3 elements.

4.4 Element Spacing of the Sensor Array

Figures 6(a) and 6(b) shows the MVDR spectrum for an element spacing of d=0.25λ and d=0.5λ, respectively. When the elements of the sensor array are placed too close to each other, mutual coupling effects dominate resulting in inaccuracies in the estimated angles of arrival, as shown in Figure 6(a) for which d=0.25λ. Mutual coupling effects for closely spaced elements must, therefore, be taken into account when designing the sensor array. To overcome this problem, the spacing between the elements of the sensor array must be increased resulting in a better resolution of the estimated peaks, as shown in Figure 6(b) for which d=0.5λ.

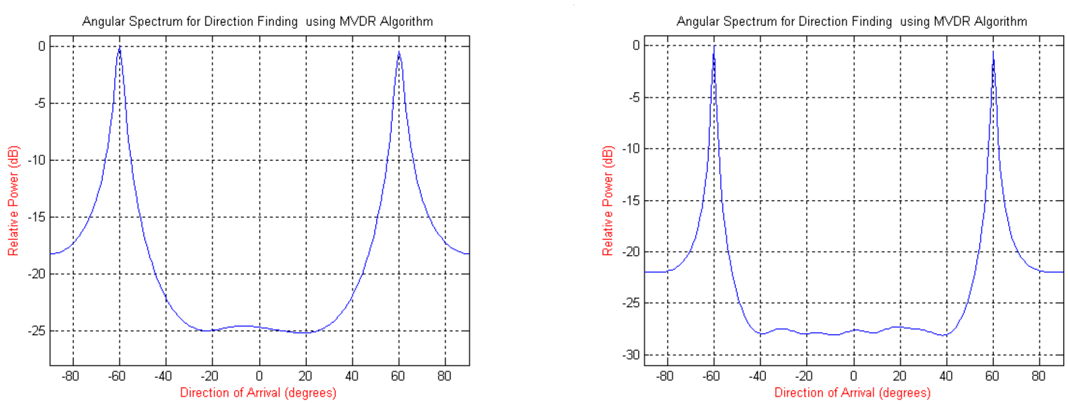


Figure 6: Effect of element spacing d on MVDR angular spectrum: (a) d= 0.25λ, and (b) d=0.5λ.

CONCLUSION

A versatile simulation tool that implements the MVDR algorithm for DOA estimation was developed together with a user-friendly GUI. A number of numerical experiments were conducted to investigate the effect of various parameters on the performance of the MVDR algorithm and its ability to resolve incoming signals accurately and efficiently. The developed simulation tool can be used to improve and accelerate the design of wireless networks. It can also be used for computer-aided learning of modern communication systems utilizing adaptive antenna arrays.

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